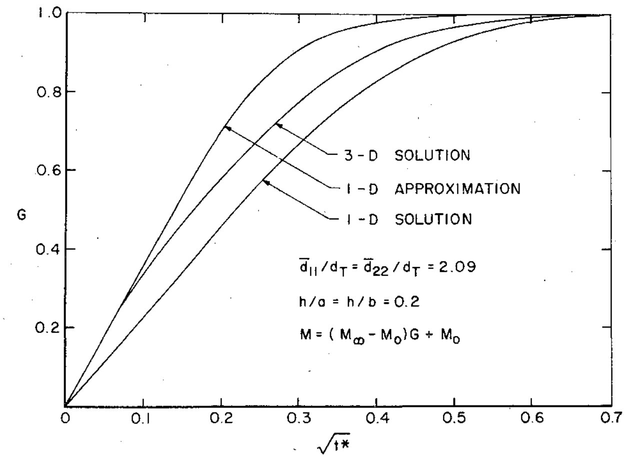

Fig. 1 Total weight gain for three-dimensional diffusion in a thick composite laminate.

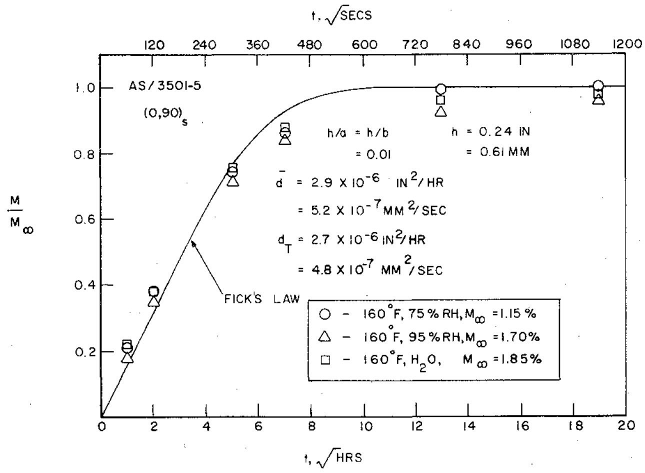

Fig. 2 Comparison of theory and experiment for a thin graphite/epoxy composite laminate.

underestimates the diffusion process because of the edge effects.

In order to assess the accuracy of the three-dimensional form of Fick's Law, two laminated plates were fabricated from Hercule's AS/3501-5 graphite/epoxy pre-preg system. One panel was a four-ply $(0,90)_s$ construction from which 2×2 in. specimens were cut. The second panel was constructed of 20 plies with the stacking geometry $(0,90)_{5s}$ from which 1-in. × 1-in. specimens were cut. Thus, the first panel provided thin specimens for measuring $\bar{d}_{33}$, while the second panel provided thick specimens for measuring three-dimensional diffusion. A number of specimens were placed on racks in environmental chambers with the temperature maintained at 160°F and the relative humidity controlled at either 75% or 95%. The remaining specimens were placed in distilled water in the environmental chamber having 85% RH and a 160°F temperature. All specimens were dried out in a vacuum oven at 200°F and weighted prior to environmental conditioning. Specimens were withdrawn at various times from the environmental chamber and their wet weight measured and recorded.

A fit of the experimental data to Fick's Law for the thin specimens is displayed in Fig. 2 where percent weight gain is plotted against $\sqrt{t}$. A small edge correction factor of about 6% was used to determine d_T. Thus, for all practical purposes, the diffusion process for these laminates was one-dimensional in nature. The value of d_T determined in Fig. 2 was used in conjunction with Eq. (16) and various solutions of Fick's Law to predict the precent weight gain as a function of $\sqrt{t}$ for the thick specimens. Comparisons between theory and experiment are shown in Fig. 3. Each data point in both Figs. 2 and 3 is an average of 3 to 5 specimens.

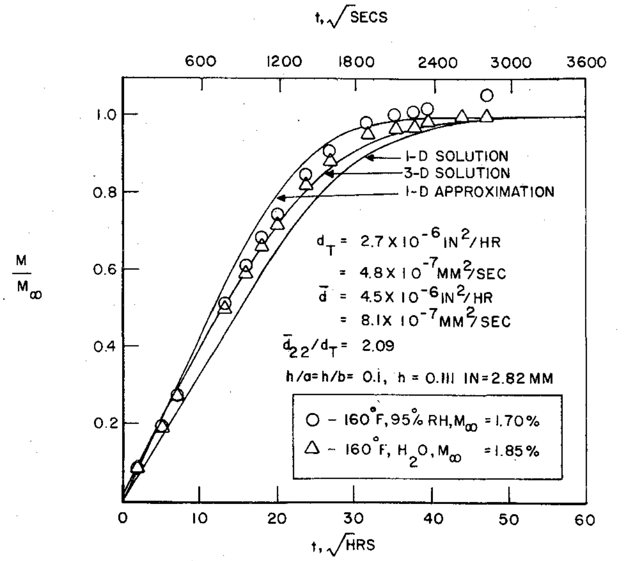

Fig. 3 Comparison of theory and experiment for a moderately thick graphite/epoxy laminate.

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Similar Solutions for an Axisymmetric Laminar Boundary Layer on a Circular Cylinder

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SIMILAR solutions of the equations describing steady, constant-density, constant-viscosity flow in laminar boundary layers are well documented for plane two-dimensional flow.[1-5] A further example of some interest is that for axisymmetric flow on a circular cylinder. Along a continuous, stationary cylindrical surface, x is measured in the direction of the generators, and, perpendicular to this, a curvilinear coordinate z is drawn on the surface. A coordinate y is measured into the flow along straight lines normal to the surface. The curvature of the surface in the y-z plane is $K(z)$ (convex positive). Velocity components in the x and y directions are, respectively, u and v; there is no velocity in the z direction.

Of the complete Navier-Stokes equations,[6] that for the x direction is differentiated with respect to y and then subtracted from the x derivative of the equation for the y direction. This yields a single equation of motion from which

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pressure and body forces have been eliminated. No approximations whatever are made.

A stream function ψ is introduced defined by

$$u=a^{-1}\psi_y, \quad v=-a^{-1}\psi_x$$

where $a=1+Ky$ (>0), and letter suffixes denote partial differentiation. By analogy with the corresponding case of plane two-dimensional flow,[3] we then eliminate 1) all derivatives higher than first order with respect to x and/or z and all products of first-order derivatives with respect to these variables; and 2) in the bracket multiplied by kinematic viscosity ν, all derivatives with respect to x and/or z.

We then introduce the dimensionless parameters $Re=U_\infty L/\nu$, $\eta=y(Re)^{1/2}/Lg$, $f=\psi(Re)^{1/2}/LU_0g$, and $\Omega=LgK/(Re)^{1/2}$, where L represents a characteristic length of the system, U_∞ a suitable mainstream reference velocity, and $g(x,z)$ the scale factor for y. The velocity $U_0(x,z)$ in the x direction is that which would be found at the boundary surface if the actual mainstream velocity distribution continued to the surface.

Since, for flow over a curved surface, similar solutions imply geometric similarity between different positions on the surface, the boundary-layer thickness at a given position must bear a fixed ratio to the radius of curvature $(=K^{-1})$ there. That is, $gK=$ const, whence $\Omega=$ const. Substitutions are made for ψ, y, and K in terms of the appropriate dimensionless parameters; f is assumed to be a function of η only, and Ω is taken as constant. Hence,

$$\alpha f(-3a^{-2}\Omega^2 f'+3a^{-1}\Omega f''-f''')+f'\left[\left\{3a^{-2}\Omega^2\eta\left(\frac{\alpha-\gamma}{2}\right)\right.\right.$$
$$\left.\left.-a^{-1}\Omega\left(\frac{\alpha+\gamma}{2}\right)\right\}f'+\left\{\gamma-2a^{-1}\Omega\eta\left(\frac{\alpha-\gamma}{2}\right)\right\}f''\right]$$
$$=-3a^{-2}\Omega^3 f'+3a^{-1}\Omega^2 f''-2\Omega f'''+af'''' \tag{1}$$

The primes denote differentiation with respect to η. Also,

$$\alpha=(Lg/U_\infty)(U_0 g)_x \tag{2a}$$

$$\gamma=(Lg^3/U_\infty)(U_0/g)_x \tag{2b}$$

where suffix x denotes partial differentiation with respect to x. For similar solutions, α and γ must be constant. If $K\neq 0$, then $g_x=0$ because K is independent of x. Thus,

$$(2LU_0/U_\infty)gg_x=\alpha-\gamma=0 \tag{3}$$

and so Eq. (1) becomes

$$\alpha f(-3a^{-2}\Omega^2 f'+3a^{-1}\Omega f''-f''')+\alpha f'(-a^{-1}\Omega f'+f'')$$
$$=-3a^{-2}\Omega^3 f'+3a^{-1}\Omega^2 f''-2\Omega f'''+af'''' \tag{4}$$

The no-slip requirement at the solid surface gives $u=0$ when $y=0=\eta$, i.e.,

$$f'(0)=0 \tag{5}$$

Also, provided that $\alpha\neq 0$,

$$\zeta=\alpha f(0) \tag{6}$$

where $\zeta=-v_0 g(Re)^{1/2}/U_\infty$, and v_0 represents the velocity through the surface. It may be shown readily[7] that, for zero mainstream vorticity,

$$f'\to a \text{ as } \eta \text{ becomes large} \tag{7}$$

and that $(U_0)_z=0$. Hence, from Eq. (2), $g_z=0$, and so, from the definition of Ω, the surface must be that of a circular cylinder.

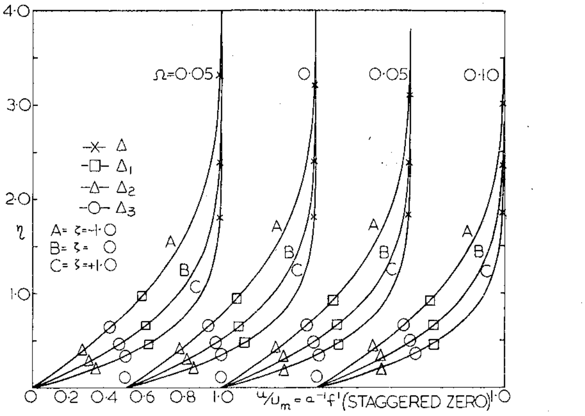

Fig. 1 Variation of tangential velocity ratio for different values of Ω and ζ.

Using condition (7), Eq. (4) may be integrated to yield

$$a^2 f'''-a\Omega f''+\Omega^2 f'+\alpha\{a^3+f(af''-\Omega f')-a(f')^2\}=0 \tag{8}$$

and it may be shown that every term omitted in the development of Eq. (8) is exactly zero, and so the equation is exact. Furthermore, there is no loss of generality in setting $|\alpha|=1$ provided that $\alpha\neq 0$. From Eqs. (2) and (3), $(U_0)_x=\alpha U_\infty/Lg^2$, and since also $g_x=0$, Eq. (8) describes flow in a layer of uniform thickness under a mainstream for which U_0 varies linearly with x.

It may be shown[7] that no monotonic velocity profiles are possible for $\alpha<0$. The equation has been solved numerically for $\alpha=1$ by an iterative technique[1] for various values of the suction parameter ζ. Figure 1 shows profiles of the ratio that the velocity component parallel to the surface bears to the component which would be found if the mainstream velocity distribution obtained throughout. (Results for $\Omega=0$ are from Terrill.[8]) The dimensionless form $\Delta=\delta(Re)^{1/2}/Lg$ of the conventional boundary-layer thickness is indicated on the curves; so are values of dimensionless displacement, momentum, and energy thicknesses defined, respectively, by

$$\Delta_1+\frac{1}{2}\Omega\Delta_1^2=\int_0^\infty (a-f')\,d\eta$$

$$\Delta_2+\frac{1}{2}\Omega\Delta_2^2=\int_0^\infty \{f'-a^{-1}(f')^2\}\,d\eta$$

$$\Delta_3+\frac{1}{2}\Omega\Delta_3^2=\int_0^\infty \{f'-a^{-2}(f')^3\}\,d\eta$$

Figure 2 shows $f''(0)$ [proportional to local skin-friction coefficient $c_f=\mu(u_y)_{y=0}/\frac{1}{2}\rho U_0^2$], and the diagram also incorporates results for $\alpha-\gamma=0$ in plane two-dimensional flow.[1,4] For a given value of ζ [here equal to $(v_0/U_0)(U_0x/\nu)^{1/2}$] in the range investigated, the effect of transverse curvature on $f''(0)$ is only about one-fourth that of longitudinal curvature and is of opposite sign. This fits with the findings of Seban and Bond,[9] who studied the effect of transverse curvature under a zero pressure gradient. Their conditions did not yield similar solutions, but they too found that the skin-friction coefficient is increased by positive (i.e., convex) transverse curvature. The effect of curvature on c_f is influenced little by the value of the suction parameter.

The particular case $\alpha=0$ yields the algebraic solution[7]

$$f'=a\{1-a^{-\zeta/\Omega}\}=a\{1-a^{v_0/K\nu}\}$$

but the mainstream conditions can be met only when $\Omega>0$ (i.e., flow outside the cylinder) and $\zeta>0$ (i.e., suction). Since

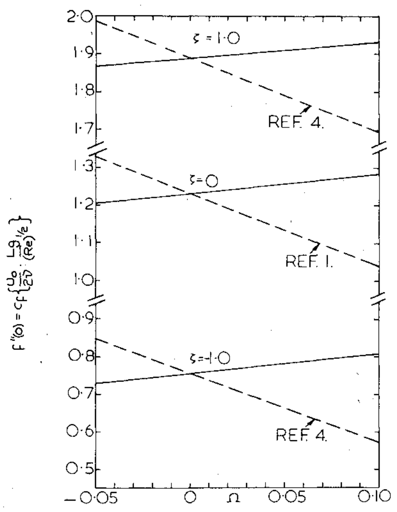

Fig. 2 Variation of $f''(0)$ with Ω and ζ.

in directions parallel to the boundary there is no variation of either thickness or velocity, this solution is not merely "similar" but "identical."

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Numerical Solution for Viscous Transonic Flow

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Introduction

THE near-inviscid supercritical flow over a thin airfoil is calculated using a second-order algorithm for the viscous transonic equation. Type-differencing, shock-point, and parabolic operators are unnecessary in the present approach; good agreement is found with the results of Martin[1] using a Murman-Cole scheme. Some special advantages of the present formulation are discussed.

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Analysis

The computation of steady, inviscid supercritical transonic flow over airfoils is complicated by the appearance of mixed supersonic/subsonic regions separated by unknown sonic lines and shocks. General existence and uniqueness theorems are still unavailable, but a number of successful schemes have been developed recently to handle mixed-type partial differential equations. A number of the methods may be considered "embedding methods" and generally involve computations in a function space other than the physical, the generalized solution of which reduces to the physical solution in some limit. One example is Garabedian's[2] "method of imaginary characteristics," where analytic continuation transforms an unstable elliptic Cauchy problem into a stable hyperbolic one in a complex space, thereby stabilizing the marching procedure. Magnus and Yoshihara,[3] for example, use the method of characteristics to calculate the steady asymptote of the unsteady hyperbolic Euler equations. Still another method is Rubbert's[4] method of parametric differentiation, in which the nonlinear equations are embedded in a parameter space where the governing equations are linear.

The most popular approach is due to Murman and Cole[5] employing type-differencing. Subsonic points are represented by central differences and supersonic points by backward differences, properly accounting for domains of influence and dependence. The manner in which grid points are type-tested is crucial, since divergence in the relaxation scheme is possible. Also, the inviscid finite-difference equations must be in proper conservation form, so that captured shocks with the correct jumps appear. The procedure relies on special parabolic and shock-point operators applied at sonic lines and shocks, and high-order truncation terms must be diffusive. It is clear that sophisticated program logic is required.

A simpler approach is to deal directly with the high-order viscous problem. Consider, for example, Burger's equation, $uu_x = \epsilon u_{xx}$, written in stationary coordinates. Since $(\frac{1}{2}u^2)_{x=a}^{x=b} = (\epsilon u_x)_a^b$, shocklike solutions with vanishing gradients at $x=a,b$ imply straightforwardly the jump conditions $u^2(a) = u^2(b)$. On the other hand, suppose that $\epsilon = 0$ identically, as in the inviscid problem. Then, $uu_x = 0$ can be multiplied by any power of u, with the result that $(u^n)_x = 0$. This admits an infinity of jump conditions, so that one or more entropy conditions must be invoked to insure uniqueness. The jump conditions, of course, are not ambiguous; they are determined by the full, high-order problem.

Similarly, the complicated logic schemes involved in inviscid Murman-Cole-type algorithms can be avoided by embedding the low-order equation in a high-order viscous system. For small-perturbation flows, the vorticity generation within the flow can be ignored, so that the inviscid velocity potential can be used. However, it is known that inviscid theory is not completely sufficient. For example, it inadequately describes the flow near the throat of a converging-diverging nozzle during the transition from the Taylor type of flow to subsonic-supersonic Meyer flow. It also is obvious that real dissipative effects must be important in narrow shock layers. To resolve the physical details near these two kinds of turning points, the usual inviscid derivation must be reconsidered to determine the circumstances under which high-order streamwise derivatives, multiplied by viscosity, are important. This special limiting process was investigated by Sichel,[6] and the result is a "viscous transonic equation" that contains a third derivative term in the disturbance velocity potential with a small coefficient that accounts for the effects of compressive viscosity in shock regions. The modified equation still describes inviscid flow only, but it implicitly contains the correct jump conditions. If accurate solution is possible, all of the salient physical features of flows developed